

Competition between ferro-retrieval and anti-ferro orders in a Hopfield-like network model for plant intelligence

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Abstract

We introduce a simple cellular-network model to explain the capacity of the plants as memory devices. Following earlier observations (Bose [1] and others), we regard the plant as a network in which each of the elements (plant cells) are connected via negative (inhibitory) interactions. To investigate the performance of the network, we construct a model following that of Hopfield, whose energy function possesses both Hebbian spin glass and anti-ferromagnetic terms. With the assistance of the replica method, we find that the memory state of the network decreases enormously due to the effect of the anti-ferromagnetic order induced by the inhibitory connections. We conclude that the ability of the plant as a memory device is rather weak.

Key words: neural networks, spin glasses, associative memories

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1 Introduction

Since the pioneering work by an Indian scientist J. C. Bose [1], plants have been regarded as networks which are capable of intelligent responses to environmental stimuli. For example, the dodder coil, which is a plastic plant, explores new host tree within hours after their initial touch contact [2]. This sort of behavior might be regarded as *plant intelligence*. If that is the case, does

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the plants compute, learn or memorize various spacial and temporal patterns in different environments as computer or our brain does ?

Recently, Peak et. al. [4] pointed out that the plants may regulate their uptake and loss of gases by distributed computation. As well known, the ability of neural networks, which is a mathematical model of brain, is also based on parallel and distributed computation. Therefore, similarities between neural network model of brains and the plant network should be discussed. Although the behavior of the dodder coil we mentioned above is due to emergence of the intelligence as a *macroscopic function*, it is important for us to investigate its *microscopic reason*.

Almost eighty years ago, Bose [1] detected electrical signaling between plants cells. Since his experiments, many examples of cross-talk, namely, the biochemical signaling pathways in plants have been found. Especially, a Boolean representation of the networks of signaling pathways is possible in terms of logical gates like AND, OR and XOR etc. These Boolean descriptions make it possible to draw analogies between plant networks and neural network models.

Recently, Brüggemann et. al. [5] found that the plant vacuolar membrane current-voltage characteristics is quite nonlinear and almost equivalent to that of a Zenner diode. Inspired by such observations, Chakrabarti and Dutta [6] identified the cell membranes as the *two state neurons* of the plants and utilized such a threshold behavior of the plant cell membranes to develop or model gates for performing simple logical operations. They found that the plant network connections are all positive (*excitatory*) or all negative (*inhibitory*), compared to the randomly positive-negative distributed synaptic connections in real brains. As a result, the plant network does not involve any frustrations in their computational capabilities. Hence, although the logical gates for computations could be achieved by such networks of plant cells, for *intelligence* of such networks they must also possess some memory capacity (in order to compare and optimise).

With this fact in mind, in this paper, we investigate the equilibrium properties of the Hopfield model in which both ferromagnetic retrieval and anti-ferromagnetic terms co-exist. The strength of the anti-ferromagnetic order is controlled by a single parameter λ . Within the replica symmetric calculation, we obtain phase diagrams of the system. The λ -dependence of the optimal loading rate α_c at zero temperature ($T = 0$) is discussed.

This paper is organized as follows. In the next section, we introduce several experiments and observations about the current-voltage characteristics of the plant cell membrane. In section 3, we model the plant with such properties by using a Hopfield-like network model in which both ferro-retrieval and anti-ferro terms exist. In section 4, we analyze the model with assistance with replica

method. In this section, we investigate to what extent the ferro-magnetic retrieval order remain against the anti-ferro disturbance. We also investigate the result of the ferromagnetic disturbance. In the final section is summary.

2 The I-V characteristics of cell membranes

In this section, we briefly mention several results concerning properties of the plant units, namely, current (I) - voltage (V) characteristics of their cell membrane. In Fig. 1, we show the typical non-linear I-V characteristics of cell membranes for the logical gates.

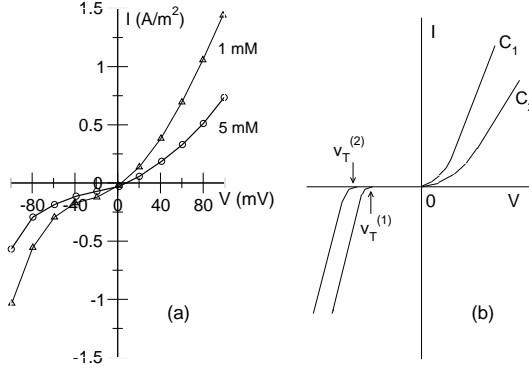


Fig. 1. (a) The non-linear I-V characteristics of cell membranes (cf. [5]). (b) Its Zener diode like representation with threshold voltages denoted by v_T .

From this figure, we find that the I-V characteristics is equivalent to that of Zenner diode. From the view point of input-output logical units like perceptrons for neural networks, the output of i th unit O_i is given by [6]

$$O_i = \Theta \left(\sum_{j=1}^N w_{ij} I_j - \theta \right); \quad \theta \equiv v_T, \quad (1)$$

where the connections w_{ij} are all positive or all negative. In the Hopfield model they are \pm randomly distributed weight matrix as given by the Hebb rule. From these experimental results and simple observations, we now have a natural question: could the plants act as memory devices as a real brain does? Obviously, in the above definition of a single unit of the plant network, there is no frustration as in the animal brains. Thus, a scope of this paper is to make clear this problem, that is to say, to what extent, this kind of limitations for sign of the weight matrix influences the ability of pattern retrieving as associative memories.

For this purpose, we introduce a simplest plant intelligence model based on a Hopfield-like model in which ferromagnetic retrieval and anti-ferromagnetic ordered phases co-exists. In the next section, we explain its details.

3 The plant intelligence model

We start from the following Hamiltonian :

$$\mathcal{H} = \frac{1}{N} \sum_{ij} \left(\lambda - \sum_{\mu=1}^p \xi_i^\mu \xi_j^\mu \right) S_i S_j \equiv \mathcal{H}_{\text{AF}} + \mathcal{H}_{\text{FR}} \quad (2)$$

$$\mathcal{H}_{\text{AF}} \equiv \frac{\lambda}{N} \sum_{ij} S_i S_j, \quad \mathcal{H}_{\text{FR}} \equiv -\frac{1}{N} \sum_{ij\mu} \xi_i^\mu \xi_j^\mu S_i S_j \quad (3)$$

where $\boldsymbol{\xi}^\mu = (\xi_1^\mu, \dots, \xi_N^\mu)$ is μ th embedded pattern and $\boldsymbol{S} = (S_1, \dots, S_N)$ means neuronal states. A single parameter λ determines the strength of the anti-ferromagnetic order, that is to say, in the limit of $\lambda \rightarrow \infty$, the system is completely determined by H_{AF} . On the other hand, in the limit of $\lambda \rightarrow 0$, the system becomes identical to the conventional Hopfield model. The purpose of this paper is to investigate the λ -dependence of the system, namely, to study the λ -dependence of the optimal loading rate $\alpha_c(\lambda)$ at $T = 0$ by using the technique of statistical mechanics for spin glasses.

4 Replica symmetric analysis

In order to evaluate macroscopic properties of the system, we first evaluate the averaged free energy :

$$\ll \log Z \gg_{\boldsymbol{\xi}} = \ll \log \text{tr}_{\{S\}} e^{-\beta H} \gg_{\boldsymbol{\xi}} = \lim_{n \rightarrow 0} \frac{\ll Z^n \gg_{\boldsymbol{\xi}} - 1}{n}. \quad (4)$$

where $\ll \dots \gg$ means the quenched average over the $p = N\alpha$ patterns. To carry out this average and spin trace, we use the replica method [9,10].

After standard algebra [9,10], we obtain the pattern-averaged replicated partition function as follows.

$$\ll Z^n \gg_{\boldsymbol{\xi}} = \prod_{\alpha, \mu} \int_{-\infty}^{\infty} \frac{dM_{\mu}^{\alpha}}{\sqrt{2\pi/N\beta}} \int_{-i\infty}^{+i\infty} \frac{dm_{\alpha}}{i\sqrt{2\pi/N\beta\lambda}} \quad (5)$$

$$\times \int_{-\infty}^{\infty} dq_{\alpha\beta} \int_{-i\infty}^{+i\infty} \frac{dr_{\alpha\beta}}{2\pi i} \exp[-Nf(\mathbf{m}, \mathbf{q}, \mathbf{M}, \mathbf{r})]. \quad (6)$$

By assuming the replica symmetric ansatz, namely, $M_{\mu}^{\alpha} = M$, $m_{\alpha} = m$, $q_{\alpha\beta} = q$, $r_{\alpha\beta} = r$, we obtain the free energy density per replica number n as follows.

$$\begin{aligned} \frac{f(m, q, M, r)}{n} &= \frac{\beta}{2} M^2 - \frac{\beta\lambda}{2} m^2 + \frac{\alpha\beta^2 r}{2} (1 - q) \\ &+ \frac{\alpha}{2} \left\{ \log[1 - \beta(1 - q)] - \frac{\beta q}{1 - \beta(1 - q)} \right\} \\ &- \log \int_{-\infty}^{\infty} Dz \log 2 \cosh \beta(\lambda m + \sqrt{\alpha r} z + M) \end{aligned} \quad (7)$$

where we defined $Dz \equiv dz e^{-z^2/2} / \sqrt{2\pi}$. We should keep in mind that physical meanings of m and M are magnetization of the system, overlap between the neuronal state \mathbf{S} and a specific recalling pattern $\boldsymbol{\xi}^1$ among αN embedded patterns, respectively. q means spin glass order parameters.

In the next section, we evaluate the saddle point of this free energy density f and draw phase diagrams to specify the pattern retrieval properties of the system.

5 Phase diagrams

In this section, we investigate the phase diagram of the system by solving the saddle point equations.

Saddle point equations: By taking the derivatives of f with respect to M, m, r and q , we obtain the saddle point equations.

$$M = \int_{-\infty}^{\infty} Dz \tanh \beta[(1 - \lambda)M + z\sqrt{\alpha r}] = -m \quad (8)$$

$$q = \int_{-\infty}^{\infty} Dz \tanh^2 \beta[(1 - \lambda)M + z\sqrt{\alpha r}] \quad (9)$$

$$r = \frac{q}{[1 - \beta(1 - q)]^2} \quad (10)$$

We solve the equations numerically to obtain the phase diagram.

T = 0 noise-less limit: We first investigate the $T = 0$ limit. In this limit, obviously, $q \rightarrow 1$. After some algebra, we find that the optimal loading rate α_c is determined by the point at which the solution of the following equation with respect to y vanishes.

$$y \left\{ \sqrt{\alpha} + \sqrt{\frac{2}{\pi}} (1 - \lambda) e^{-\frac{y^2}{2}} \right\} = (1 - \lambda) \{1 - 2H(y)\} \quad (11)$$

where $H(x)$ is defined by $H(x) = \int_x^\infty Dz$. In Figure 2, we plot the optimal

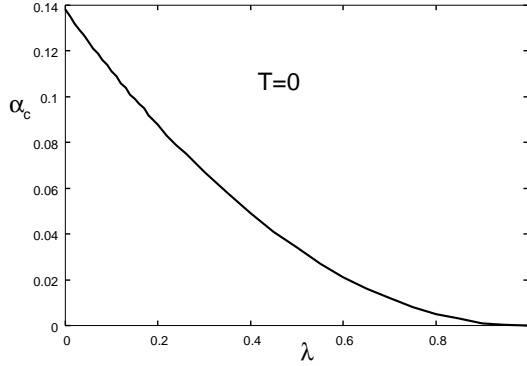


Fig. 2. The optimal loading rate α_c as a function of λ . $\alpha_c(\lambda)$ decreases monotonically.

loading rate α_c as a function of λ . From this figure, we see that the optimal loading capacity $\alpha_c(\lambda)$ monotonically decreases to zero as $\sim (1 - \lambda)^2$. This means that the ferro-magnetic retrieval order was destroyed by adding the anti-ferromagnetic term to the Hamiltonian. Thus, we conclude that if the weight matrix of the networks is all positive, the plant intelligence model does not act as a memory device.

Spin Glass-Para phase boundary: Before we solve the saddle point equations for $T \neq 0$, it should be important to determine the phase boundary between the spin glass and para-magnetic phases. The phase transition between these two phases is first order, by expanding the saddle point equations around $M = q = 0$, we obtain

$$q \simeq \beta^2 \alpha r \int_{-\infty}^{\infty} z^2 Dz = \beta^2 \alpha \frac{q}{(1 - \beta)^2}. \quad (12)$$

By solving this equation with scaling $\beta \rightarrow (1 - \lambda)\beta$ and $T = \beta^{-1}$, we obtain the phase boundary line :

$$T_{SG} = (1 - \lambda)(1 + \sqrt{\alpha}) \quad (13)$$

Phase diagrams for $T \neq 0$: Here, we investigate the phase diagram for $T \neq 0$ by solving the saddle point equations (8),(9) and (10) numerically. From

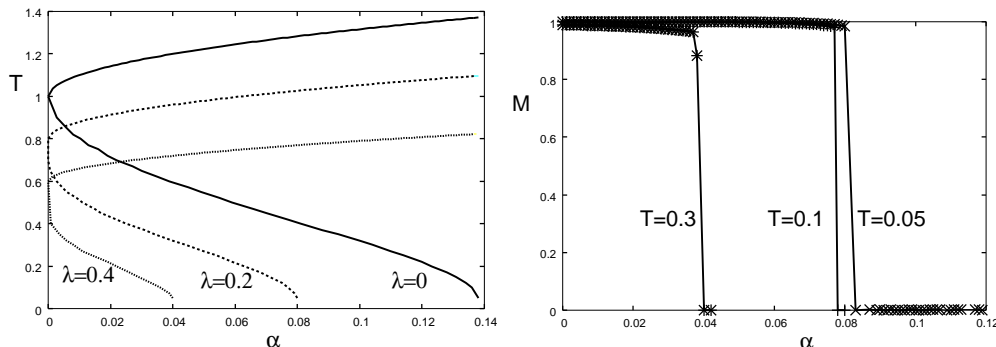


Fig. 3. The phase diagram of the system. Ferro-retrieval phase shrinks to zero as λ increases to 1. The para-spinglass boundary is analytically obtained as $T_c = (1 - \lambda)(1 + \sqrt{\alpha})$ and is independent of λ .

Fig. 4. The overlap M as a function of α for the case of $\lambda = 0.2$ at temperatures $T = 0.05, 0.1$ and $T = 0.3$.

this figure, we find that the ferro magnetic retrieval phase shrinks to zero as the anti-ferro magnetic order increases, namely, $\lambda \rightarrow 1$. The behavior of the overlap M as a function of α is shown in the right panel in Figure 3. The overlap M becomes zero discontinuously at $\alpha = \alpha_c$.

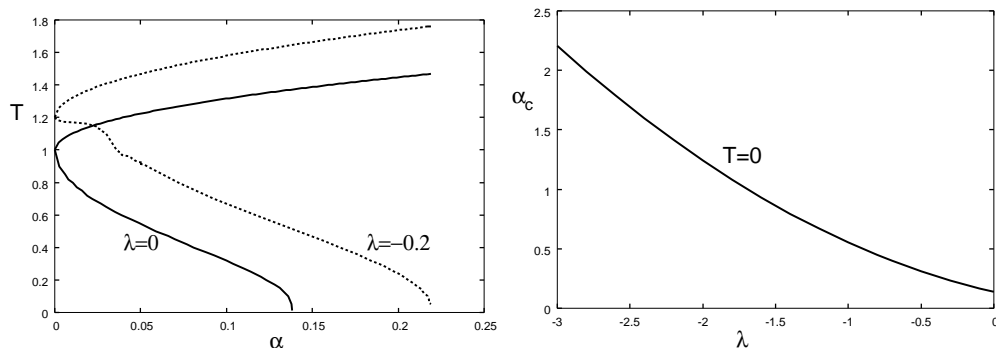


Fig. 5. The phase diagram of the system for negative $\lambda = -0.2$.

Fig. 6. The optimal loading rate α_c as a function of $\lambda (< 0)$ at $T = 0$.

Negative λ case: Here, we consider the case of negative λ . From the Hamiltonian, we find

$$\mathcal{H} = -\frac{1}{N} \sum_{ij} \sum_{\mu=1}^p \xi_i^\mu \xi_j^\mu S_i S_j - \frac{\lambda'}{N} \sum_{ij} S_i S_j; \quad \lambda' = -\lambda (> 0). \quad (14)$$

When λ increases, the system changes to the pure ferromagnet. Let us think about the limit of $\lambda \rightarrow -\infty$ in the saddle point equation (8). Then, the

term $(1 - \lambda)M$ appearing in the argument of $\tanh[\beta(\cdot \cdot \cdot)]$ becomes dominant, namely, $(1 - \lambda)M \gg z\sqrt{\alpha r}$ even if the loading rate α is large. Consequently, equation (8) leads to

$$M \simeq \int_{-\infty}^{\infty} Dz \tanh[\beta(1 - \lambda)M] = \tanh[\beta(1 - \lambda)M]. \quad (15)$$

If the factor $1 - \lambda$ is large enough, the term $\tanh[\beta(1 - \lambda)M]$ becomes $\text{sgn}[\beta M]$ and saddle point equation (8) leads to $M = \text{sgn}[\beta M]$. Apparently, this equation has always a positive solution even if the temperature $T = \beta^{-1}$ is large. In this sense, the factor $(1 - \lambda)$ has a meaning of temperature re-scaling. It is also possible for us to understand this result from the different point of view. In the saddle point equation :

$$M = \int_{-\infty}^{\infty} Dz \tanh \beta[(1 - \lambda)M + z\sqrt{\alpha r}]; \quad (16)$$

the second term appearing in the argument of \tanh , $z\sqrt{\alpha r}$ means *cross-talk noise* from the other patterns ξ^μ , ($\mu = 2, \dots, p$) and obeys Gaussian distribution $e^{-z^2/2}/\sqrt{2\pi}$. On the other hand, the first term $(1 - \lambda)M$ represents *signal* of the retrieval pattern ξ^1 . Therefore, if the second term $z\sqrt{\alpha r}$ is dominant, the system cannot retrieve the embedded pattern ξ^1 . Usually, r in the second term grows rapidly as T increases. And obviously, if α increases, the noise term $z\sqrt{\alpha r}$ also increases. As a result, the signal part $(1 - \lambda)M$ becomes relatively small and the system moves from the retrieval phase to the spin glass phase. However, if λ is negative large, the signal part is dominant and the noise part becomes vanishingly small. This is an intuitive reason why the optimal loading rate increases for negative λ . In Figure 6, we plot the optimal loading rate α_c as a function of $\lambda (< 0)$ at $T = 0$. As we mentioned already, the optimal loading rate α_c monotonically increases as λ goes to $-\infty$.

6 Summary

With the identification [6] of plant cell membranes as *neurons*, and demonstrating [6] the possibility of logical gate operations in these cellular networks, the discussion on *intelligence* of a plant could not be complete without the demonstration of memory in such, either excitatorily or inhibitorily connected, cellular networks. In this paper, we introduce a simple model based on a Hopfield-like network for explaining the memory capacity of the plant cell network. Following the experimental observations [5], we construct a Hopfield model in which ferromagnetic retrieval and anti-ferromagnetic terms co-exist.

The strength of disturbance of pattern retrieval by the anti-ferro order is controlled by a single parameter: one can easily see that the internal fields h_{AF} and h_{FR} contributed by \mathcal{H}_{AF} and \mathcal{H}_{FR} respectively are of order λ and $\sqrt{pN}/N = \sqrt{\alpha}$ in (3). We find that the anti-ferromagnetic order prevents the system from recalling a pattern. However, even when λ is greater than $\sqrt{\alpha}$, the network still possess considerable memory capacity (see Fig. 2). This result means that the ability of the plant as a memory device is rather weak, if we set all weight connections to positive values. Our analysis here has been for fully connected networks. For real plants, the cell membranes should be located on a finite dimensional lattice [11] or on a scale-free network [12]. Investigations for these situations will be made in future.

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